E 1358 (Pages : 3) Reg. No.....

Name.....

C.B.C.S.S. – B.Sc. DEGREE EXAMINATION, APRIL 2011

Second Semester

Core Course—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for Model I and Model II, B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
A bunch of 4 *questions has weight* 1.

I. 1 What is the x co-ordinate of the point of intersection of the tangents of 't_1' and 't_2' on the parabola $y^2 = 4ax$?

2 What is the slope of the normal at 't' on the parabola $y^2 = 4ax$?

3 What is the maximum number of normals that can be drawn from a given point to an ellipse ?

4 Define conjugate diameters of an ellipse.

II. 5 What is the eccentricity of a rectangular hyperbola ?

6 If the eccentricity of a hyperbola is $\sqrt{2}$, what is the angle between its asymptotes ?

7 What is the equation of the hyperbola conjugate to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?

8 What is the general equation of a line passing through the pole in polar co-ordinate system ?

III. 9 Which curve is represented by $\frac{l}{r} = 1$ in polar co-ordinates ?

10 What is the period of the complex circular function $f(x) = \cos x$?

11 What is the real part of $\sin(\alpha + \beta i)$?

12 How many real roots are there for the equation $x^5 - 1 = 0$?

IV. 13 Suppose A and B are two square matrices such that A and AB are non-singular. Is it true that B is non-singular ?

14 If a system of 'n' non-homogeneous equations in 'n' unknowns has a unique solution, then what is the rank of the coefficient matrix ?

Turn over

15 How many distinct characteristic roots are there for the $n \times n$ identity matrix ?

16 Write a 3×3 matrix with characteristic polynomial $(x-2)^2(x-3)$.

$(4 \times 1 = 4)$

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight **1.**

17 Prove that two tangents can be drawn from a given point to an ellipse.

18 Prove that the polar of the focus $(ae, 0)$ corresponding to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x=\frac{a}{e}$.

19 Find the slope of the normal to the rectangular hyperbola $xy = c^2$ at the point 't'.

20 Find the distance between the points (r_1, θ_1) and (r_2, θ_2) in polar co-ordinate system.

21 Prove that $\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$.

22 Separate into real and imaginary parts : $\sinh(\alpha+\beta i)$.

23 Give a 3×3 matrix which is equivalent to :

$$\begin{bmatrix} 1 & 4 & 9 \\ -1 & 2 & 4 \\ 3 & -5 & 7 \end{bmatrix}.$$

24 Verify Cayley-Hamilton theorem for the matrix :

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

$(5 \times 1 = 5)$

Part C (Short Essays Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Prove that if normals at two points of a parabola intersect on the curve, then the product of the ordinates of the points is $8a^2$.

26 Show that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is constant.

27 Prove that if the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ is a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$, then $(A - e)^2 + B^2 = 1$.

28 Factorize $x^7 - 1$ into linear factors.

29 Find the canonical matrix row equivalent to :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{bmatrix}.$$

30 Find all solutions of :

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ 2x_1 + 5x_2 - 2x_3 &= 3 \\ x_1 + 7x_2 - 7x_3 &= 5 \end{aligned}$$

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 Show that the locus of the poles of normal chords of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{a^6}{x^2} + \frac{b^6}{y^2} = \left(a^2 - b^2\right)^2.$$

32 Sum the series : $\frac{1}{2}\sin\alpha + \frac{1\cdot3}{2\cdot4}\sin 2\alpha + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\sin 3\alpha + \cdots$ to infinity.

33 Find the characteristics roots and the associated characteristic vectors of the matrix :

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

$(2 \times 4 = 8)$